Published by Institute of Physics Publishing for SISSA

Received: *May 3, 2005*
Accepted: *June 16, 2005*
Published: *July 20, 2005*

Quadratic α' corrections to T-duality

Ghasem Exirifard

SISSA/ISAS
Via Beirut 2-4, I-34013 Trieste, Italy, and
INFN, Sezione di Trieste, Italy
Email: exir@sissa.it

Abstract: The quadratic α' corrections to the two-dimensional black hole and to its T-dual are calculated. These backgrounds are used to write the covariant form of the α'^2 corrections to the T-duality for general time-dependent backgrounds of dilaton and diagonal metric in the bosonic string theory.

Keywords: Bosonic Strings, Black Holes in String Theory, String Duality.

Contents

1. Introduction

It is known that a chain of dualities relates different perturbative string theories on flat space-time to each other. The explicit form of possible generalization of these dualities to an arbitrary background is known only at the level of SUGRA or gravity. In general it is not known how these dualities should be modified as one takes into account the sub-leading string corrections.

T-duality [1] is the first known duality of string theory. It connects two points of the moduli space of string theory. In the LEEA this connection means that a map exists from the field contents of a given background to the field contents of the corresponding T-dual background. Truncating the LEEA of string theory to the leading term, the so-called SUGRA or gravity, gives tree level rules of this map [2]. These tree level rules should be modified as one adds the sub leading , in particular the word-sheet, corrections. The linear α'-corrections to these rules in bosonic and heterotic string theory have been computed in [3 – 6].

Discrete T-duality can be generalized to a continuous symmetry group known as $O(d, d)$ transformation [7]. The argument that the string tree level effective action is invariant under continuous duality transformations to all orders in α' is given in [8] and is generalized for heterotic string theory in [9]. Also [10, 11] provide the explicit computation that $O(d, d)$ invariance is preserved at order of α'^2 given a field redefinition and coordinate transformations. In [12] the explicit α'^2 corrections to the rules of discrete T-duality were computed for time-dependent diagonal metric and dilaton in the bosonic string. However the rule for the direction in which the T-duality is applied was provided only to linear order in α'. In this note we are going to compute the explicit form of the α'^2 corrections to this rule. The note is organized in the following way.

We first review the results of [12] and write explicitly the ambiguity of the covariant form of T-duality. Next we calculate the linear and the quadratic α' corrections to the two-dimensional black hole and to its corresponding T-dual background. We use these α'^2 corrected backgrounds to write uniquely the sought α'^2 corrections.

In the third section we discuss the behaviour of the α'^2 corrected two-dimensional black hole. In the fourth section we summarize and discuss the results.

2. The α' corrections to T-duality

The LEEA action of the free string theory is provided by the vanishing of the β function of the corresponding sigma model. The β functions of the free bosonic string theory for backgrounds of dilaton (ϕ) and metric in d dimensional space-time at the quadratic order in α' read [13, 14]

$$\begin{aligned}
\frac{1}{\alpha'}\beta_{ij} &= R_{ij} + 2\nabla_i\nabla_j\phi + \frac{1}{2}\alpha' R_{iklm}R_j{}^{klm} + \\
&\quad + \alpha'^2\Big\{\frac{1}{8}\nabla_k R_{ilmn}\nabla^k R_j{}^{lmn} - \frac{1}{16}\nabla_i R_{klmn}\nabla_j R^{klmn} + \frac{1}{2}R_{klmn}R_i{}^{mlp}\, R_j{}^{kn}{}_p - \\
&\qquad - \frac{3}{8}\, R_{iklj}\, R^{kmnp}\, R^l{}_{mnp} + \frac{1}{32}\, \nabla_j\nabla_i(R_{klmn}R^{klmn})\Big\}\,,
\end{aligned} \tag{2.1}$$

$$\begin{aligned}
\frac{1}{\alpha'}\beta_{\phi} &= \frac{d-26}{6\alpha'} - \frac{1}{2}\Box\phi + \partial_k\phi\partial^k\phi + \frac{1}{16}\alpha' R_{klmn}R^{klmn} + \\
&\quad + \alpha'^2\Big\{-\frac{3}{16}R^{kmnp}R^l{}_{mnp}\nabla_k\nabla_l\phi + \frac{1}{32}R_{klmn}R^{mnpq}R_{pq}{}^{kl} - \\
&\qquad - \frac{1}{24}R_{klmn}R^{qnpl}R_q{}^m{}_p{}^k + \frac{1}{64}\,\partial_i(R_{klmn}R^{klmn})\partial^i\phi\Big\}\,, \\
&\quad i,\ldots,q \in \{0\ldots 25\}\,.
\end{aligned} \tag{2.2}$$

where the computation is done in the dimensional regularization method in the minimal substraction scheme. These equations provide the perturbative dynamics of the metric and the dilaton around the flat space-time. Other regularization methods and renormalization schemes give different β functions. The β functions of different schemes should be related to one another by perturbative field redefinition, e.g.[1]

$$g_{\mu\nu} \to g_{\mu\nu} + \alpha'(c_1\, R_{\mu\nu} + c_2\, R\, g_{\mu\nu} + c_3\, \nabla_\mu\phi\, \nabla_\nu\phi + c_4\, \Box\, \phi\, g_{\mu\nu} + c_5\nabla_\mu\nabla_\nu\phi) + O(\alpha'^2)\,,$$

where $c_1, \ldots, c_5$ are arbitrary functions of the dilaton. If we had been interested in finding the exact solution of the truncated β functions[2] then this ambiguity would have allowed us to redefine the fields to possibly obtain an action with either some divergence free covariant currents [17] or no exact unbounded linearized mode [15]. Note that "T-duality symmetry

[1] It is the field redefinition ambiguity at the linear order in α' for schemes in which the metric is a tensor of rank two.

[2] For example [15] and [16] are interested in finding the exact solutions of the truncated β functions in a different content. However here the truncated equations do not admit any exact solution with a perturbative expansion in α'.

JHEP07(2005)047

is a well-defined notion over and above the presence of field redefinition ambiguity, in the sense that if it is present in one choice of scheme, it may be modified but will nonetheless also be present in any other scheme" [18].

One way to obtain the α' corrections to T-duality is to require that T-duality must commute with the vanishing of the β functions [4, 6]. However at the order α'^2 this method requires lengthy calculations. To avoid these calculations one can select different background and calculate their α'^2 corrections and the α'^2 corrections to their corresponding T-dual backgrounds. Requiring T-duality to map these α'^2 corrected background to each other enables one to write the corrections to T-duality. In [12] the quadratic α' corrections to the general diagonal Kasner metric were computed and the rules of T-duality in x_{25} for time-dependent backgrounds of diagonal metric and dilaton were found to be[3]

$$\begin{aligned}
&\ln \tilde{g}_{25\,25} - \frac{\alpha'}{4}\tilde{\nabla}_\mu \ln \tilde{g}_{25\,25} \cdot \tilde{\nabla}^\mu \ln \tilde{g}_{25\,25} = \\
&\qquad = -\left\{\ln g_{25\,25} - \frac{\alpha'}{4}\nabla_\mu \ln g_{25\,25} \cdot \nabla^\mu \ln g_{25\,25}\right\} + O(\alpha'^2\nabla^4)\,, \qquad (2.3)\\
&\ln \tilde{g}_{ii} + \frac{\alpha'^2}{32}\tilde{\nabla}_\mu \ln \tilde{g}_{25\,25} \cdot \tilde{\nabla}^\mu \ln \tilde{g}_{ii}\, \tilde{\nabla}_\nu \ln \tilde{g}_{25\,25} \cdot \tilde{\nabla}^\nu \ln \tilde{g}_{25\,25} = \\
&\qquad = \ln g_{ii} + \frac{\alpha'^2}{32}\nabla_\mu \ln g_{ii} \cdot \nabla^\mu \ln g_{25\,25}\, \nabla_\nu \ln g_{25\,25} \cdot \nabla^\nu \ln g_{25\,25} + O(\alpha'^3\nabla^6)\,, \qquad (2.4)\\
&\tilde{\phi} - \frac{1}{4}\ln\det\tilde{g} + \frac{\alpha'^2}{32}\,\tilde{\nabla}_\mu \ln \tilde{g}_{25\,25}\, \tilde{\nabla}_\nu \ln \tilde{g}_{25\,25}\, \tilde{\nabla}^\mu\tilde{\nabla}^\nu \ln \tilde{g}_{25\,25} = \\
&\qquad = \phi - \frac{1}{4}\ln\det g + \frac{\alpha'^2}{32}\,\nabla_\mu \ln g_{25\,25}\, \nabla_\nu \ln g_{25\,25}\, \nabla^\mu\nabla^\nu \ln g_{25\,25} + O(\alpha'^3\nabla^6)\,. \qquad (2.5)
\end{aligned}$$

where "$\tilde{g}, \tilde{\phi}$" and "g, ϕ" represent the metric and the dilaton respectively of the background and its T-dual in the co-moving frame in the string frame. The α'^2 corrections to the Kasner metric are not sufficient to write uniquely the α'^2 corrections to (2.3). For the diagonal Kasner metric in the string frame,

$$ds^2 = -dt^2 + \sum_{i=1}^{25} t^{2p_i} dx_i^2\,, \qquad (2.6)$$

$$\phi(t) = \frac{\sum p - 1}{2}\ln t\,, \qquad (2.7)$$

one notices that "(2.3)" should be modified to [12]

$$\begin{aligned}
\ln \tilde{g}_{25\,25} - \frac{\alpha'}{4}\,\tilde{\nabla}_\mu \ln \tilde{g}_{25\,25} \tilde{\nabla}^\mu \ln \tilde{g}_{25\,25} \;=\; & -\Bigg\{\ln g_{25\,25} - \frac{\alpha'}{4}\nabla_\mu \ln g_{25\,25} \nabla^\mu \ln g_{25\,25} + \\
& + \frac{\alpha'^2\, p_{25}^4}{t^4} - \frac{\alpha'^2 p_{25}^2}{t^4}\Bigg\}, \qquad (2.8)
\end{aligned}$$

There exists only one way to express p_{25}^4/t^4 as the "covariant" derivative of the dilaton and the logarithm of the Kasner metric:

$$\frac{p_{25}^4}{t^4} = \frac{1}{16}(\nabla_\mu \ln g_{25\,25}\, \nabla^\mu \ln g_{25\,25})^2\,, \qquad (2.9)$$

[3]Note that with this notation "covariant" derivative acts on the logarithm of the metric as it had been an scalar.

while there are the following five candidates for p_{25}^2/t^4,

$$A^\star = \nabla_\mu \ln g_{25\,25}\, \nabla^\mu \ln g_{25\,25}\, \nabla_\nu \left(\phi - \frac{1}{4}\ln\det g\right) \nabla^\nu \left(\phi - \frac{1}{4}\ln\det g\right), \tag{2.10}$$

$$A = \nabla_\mu \ln g_{25\,25}\, \nabla_\nu \ln g_{25\,25} \nabla^\mu \left(\phi - \frac{1}{4}\ln\det g\right) \nabla^\nu \left(\phi - \frac{1}{4}\ln\det g\right), \tag{2.11}$$

$$B = \frac{1}{2}\nabla_\mu \ln g_{25\,25}\, \nabla_\nu \ln g_{25\,25}\, \nabla^\nu\nabla^\mu \left(\phi - \frac{1}{4}\ln\det g\right), \tag{2.12}$$

$$C = \frac{1}{2}\nabla_\mu \ln g_{25\,25}\, \nabla_\nu \left(\phi - \frac{1}{4}\ln\det g\right) \nabla^\nu\nabla^\mu \ln g_{25\,25}\,, \tag{2.13}$$

$$E = \frac{1}{8}\nabla_\mu\nabla_\nu \ln g_{25\,25}\, \nabla^\mu\nabla^\nu \ln g_{25\,25}\,. \tag{2.14}$$

In direct multiplication of spaces $|\nabla(\phi - \frac{1}{4}\ln\det g)|^2$ can depend on the coordinates of the individual spaces. We assume that T-duality rules for direct multiplications of curved spaces are given by the T-duality rules of each individual space. Therefore we exclude the possibility of writing p_{25}^2/t^4 by (2.10). Note that the rest of possibilities respect this assumption and the general form of "(2.8)" must be a linear combination of them,

$$\begin{aligned}
\ln \tilde g_{25\,25} &- \frac{\alpha'}{4}\tilde\nabla_\mu \ln \tilde g_{25\,25} \tilde\nabla^\mu \ln \tilde g_{25\,25} \\
&= -\bigg\{ \ln g_{25\,25} - \frac{\alpha'}{4}\nabla_\mu \ln g_{25\,25} \nabla^\mu \ln g_{25\,25} + \frac{\alpha'^2}{16}(\nabla_\mu \ln g_{25\,25} \nabla^\mu \ln g_{25\,25})^2 - \\
&\qquad - \alpha'^2 \left(a\,A + b\,B + c\,C + e\,E\right)\bigg\},
\end{aligned} \tag{2.15}$$

where "a, b, c, e" are real numbers satisfying "$a + b + c + e = 1$". The α'^2 corrections to other backgrounds should be calculated to fix these numerical coefficients. The simplest background to be considered is a two-dimensional black hole, as a solution of the leading beta-functions of the non-critical string theory:

$$R_{\mu\nu} + 2\,\nabla_\mu\nabla_\nu\phi = 0\,, \tag{2.16}$$

$$\frac{d-26}{6\alpha'} - \frac{1}{2}\Box\phi + |\nabla\phi|^2 = 0\,. \tag{2.17}$$

Using the convention "$\alpha' = \frac{26-d}{6}$" in "d" dimensional space-time, this solution reads [19, 20]

$$\begin{aligned}
ds^2 &= dt^2 + \tanh(t)^2\, dr^2 + dx_{d-2}^2\,, \\
\phi(t) &= -\ln(\cosh(t))\,,
\end{aligned} \tag{2.18}$$

where dx_{d-2} is "$d-2$" flat directions. This solution with an appropriate periodicity ($r \sim r + 2\pi$) is a fair candidate for a two-dimensional black hole with a geometry of semi-infinite cigar [20, 21]. We assume that the non-critical string theory has the following perturbative[4]

[4] To make it perturbative, first we analytically extend the two-dimensional black hole to $d = 26 - \epsilon$ where ϵ is a sufficiently small positive number. And at the end of the calculation we analytically extend the solution back to arbitrary d.

background

$$
\begin{aligned}
ds^2 &= dt^2 + f(t)\, dr^2 + dx_{d-2}^2\,, && (2.19)\\
f(t) &= \tanh(t)^2 \left(1 + \alpha'\, f^{(1)}(t) + \alpha'^2\, f^{(2)}(t) + \cdots\right), && (2.20)\\
\phi(t) &= -\ln(\cosh(t)) + \alpha'\, \phi^{(1)}(t) + \alpha'^2\, \phi^{(2)}(t) + \cdots\,. && (2.21)
\end{aligned}
$$

Inserting these perturbative series in the beta functions gives a set of ordinary linear differential equations for "$f^{(1)}(t), f^{(2)}(t), \phi^{(1)}(t), \phi^{(2)}(t)$". These equations have unique answer for any given boundary condition. Therefore the assumptions do not contradict themselves in the sense that the set of differential equations for the α' terms is not overdetermined. We fix the boundary condition assuming that there exists no correction at infinity,

$$
f^{(1)}(\infty) = f^{(2)}(\infty) = \phi^{(1)}(\infty) = \phi^{(2)}(\infty) = 0\,. \tag{2.22}
$$

This boundary conditions results to the α'^2 corrected background

$$
\begin{aligned}
f(t) &= \tanh(t)^2 \left(1 - \frac{2\,\alpha'}{\cosh(t)^2} + \frac{\alpha'^2\,(5-\cosh(2t))}{\cosh(t)^4} + \cdots\right), && (2.23)\\
\phi(t) &= -\ln(\cosh(t)) - \frac{\alpha'}{2\,\cosh(t)^2} - \frac{\alpha'^2 \sinh(t)^2}{2\cosh(t)^4} + \cdots\,. && (2.24)
\end{aligned}
$$

In the next section we discuss in more detail the behaviour of the α' corrected metric.

We are intersted in the modification of the T-duality so we consider the following background,

$$
\begin{aligned}
d\tilde{s}^2 &= dt^2 + \frac{1}{\tanh(t)^2}\, dr^2 + dx_{d-2}^2\,, && (2.25)\\
\tilde{\phi}(t) &= -\, \ln(\sinh(t))\,, && (2.26)
\end{aligned}
$$

which is related to the two-dimensional black-hole by T-duality in the direction of r. Its α'^2 corrections follow

$$
\begin{aligned}
d\tilde{s}^2 &= dt^2 + \tilde{f}(t)\, dr^2\,, && (2.27)\\
\tilde{f}(t) &= \frac{1}{\tanh(t)^2} \left(1 + \frac{2\alpha'}{\sinh(t)^2} + \frac{\alpha'^2\,(5+\cosh(2t))}{\sinh(t)^4}\right), && (2.28)\\
\tilde{\phi}(t) &= -\, \ln(\sinh(t)) + \frac{\alpha'}{2\,\sinh(t)^2} + \frac{\alpha'^2\,\cosh(t)^2}{2\,\sinh(t)^4}\,, && (2.29)
\end{aligned}
$$

where it is assumed that the corrections vanish at infinity. The covariant form of T-duality rules maps the α'^2 corrected two-dimensional black hole to its T-dual iff

$$
\begin{aligned}
a_2 &= -c\,,\\
b &= 0\,,\\
e &= 1\,. && (2.30)
\end{aligned}
$$

We set a_2, b and c to the above values. Furthermore the leading β functions for a time-dependent background[5] implies

$$cC + a_2 A_2 = c\,(C - A_2) \sim \beta_{25\,25} = 0\,. \tag{2.31}$$

Using "(2.30)" and "(2.31)" simplifies the T-duality rule of the transverse direction to[6]

$$\begin{aligned}
&\ln\tilde{g}_{25\,25} - \frac{\alpha'}{4}\,\tilde{\nabla}_\mu \ln\tilde{g}_{25\,25}\,\tilde{\nabla}^\mu \ln\tilde{g}_{25\,25} + \frac{\alpha'^2}{32}(\tilde{\nabla}_\mu \ln\tilde{g}_{25\,25}\,\tilde{\nabla}^\mu \ln\tilde{g}_{25\,25})^2 - \\
&\qquad\qquad - \frac{\alpha'^2}{16}\tilde{\nabla}_\mu\tilde{\nabla}_\nu \ln\tilde{g}_{25\,25}\,\tilde{\nabla}^\mu\tilde{\nabla}^\nu \ln\tilde{g}_{25\,25} = \\
&= -\Big(\ln g_{25\,25} - \frac{\alpha'}{4}\,\nabla_\mu \ln g_{25\,25}\,\nabla^\mu \ln g_{25\,25} + \frac{\alpha'^2}{32}(\nabla_\mu \ln g_{25\,25}\,\nabla^\mu \ln g_{25\,25})^2 - \\
&\qquad - \frac{\alpha'^2}{16}\nabla_\mu\nabla_\nu \ln g_{25\,25}\,\nabla^\mu\nabla^\nu \ln g_{25\,25}\Big).
\end{aligned} \tag{2.32}$$

This rule beside "(2.3)" and "(2.4)" provides the quadratic α' corrections to T-duality on time-dependent backgrounds of diagonal metric and dilaton. As a check of the consistency of our procedure these rules have been checked to be true for the Schwarzschild metric in "$D = 4, 5$" dimensions and T-duality in the direction of the time-like Killing vector outside the horizon[7]. It is not necessary to check these rules on different backgrounds since the explicit computation of [11] supports the expansion of T-duality rules at order α'^2.

At this stage it is natural to ask if the α' corrections to T-duality can be canceled by an appropriate field redefinition. In other words, is there any appropriate renormalization scheme and regularization method which gives no correction to the tree level rules of T-duality? The answer is negative. One can check that even the linear α' corrections to T-duality can not be compensated by a field redefinition which leaves the tensor property of the metric intact. In the former works [10, 11] either the redefined metric had not been a tensor or the metric definitions had not been the same in both spaces. Choosing different definitions for the metrics means choosing different schemes for the background and for its T-dual. If we do so then the corrections in the space can not be directly mapped to the corrections in the T-dual space. Choosing a scheme in which the metric is not a tensor means choosing a regularization method which breaks the general covariance of the theory. In such a regularization method one must be extremely careful about interpreting the results. It is preferable to work in a scheme which respects the fundamental symmetries of the theory. In such a scheme T-duality must be modified. This argument is supported by the fact that T-duality rules are modified for the conjectured α' exact backgrounds [22].

3. The α'^2 corrected two-dimensional metric

It is known that the string coupling constant is related to the dilaton. Hence the string coupling constant receives α' corrections through the corrections to the dilaton,

$$g_s = g_0\, e^{\phi + \alpha'\phi^{(1)} + \alpha'^2\phi^{(2)} + \cdots}\,. \tag{3.1}$$

[5] When the components of the metric depend only on time.

[6] Also note that it is rewritten in the form that the square of the T-duality is the identity.

[7] The α'^2 corrections to Schwarzschild black hole and its T-dual in $D = 5$ are presented in the appendix, for $D = 4$ the results can be found in [12]

JHEP07(2005)047

Thus a small tree level string coupling constant generally does not guarantee a small string coupling constant. In the case of the two dimensional black hole one sees that the α' corrections to the string coupling constant are finite. Therefore the string coupling constant will be small if the tree level one is small. Hence the string loop effects can be consistently ignored not only at the tree level [23] but also to, at least, the quadratic order in α'.

The α' and α'^2 corrected metrics become zero at some positive values of t near "$t = 0$". These zeros correspond to intrinsic singularities since their corresponding Ricci scalars diverge. A similar behavior has been reported in [24] where AdS/CFT correspondence is used to calculate specific quantum corrections to the general smooth half BPS solutions of type-IIB supergravity [25] and it is observed that the quantum corrections can change the coordinate singularity to either an intrinsic singularity or a highly curved geometry.

However the singularities of the two-dimensional black hole occur at the region where the tree level curvature is of the order of the string length. Thus no concrete conclusion can be achieved unless one takes into account all the α' corrections. It should be useful to generalize [27, 28] toward the analog of the DBI action [26] for gravity coupled to dilaton. In the perturbative studies if one insists on having a smooth α' corrected two-dimensional black hole then one can choose either of the following methods,

1. The α' corrected metric could be smooth in a different coordinate system. For example one can write the metric in the following form

$$ds^2 = \left(1 + \frac{2\alpha'}{\cosh(t)^2} + \frac{2\sinh(t)^2\alpha'^2}{\cosh(t)^4} + O(\alpha'^3)\right) dt^2 + \tanh(t)^2\, dr^2 + dx^2_{d-2}\,, \tag{3.2}$$

   In this new set of coordinates one does not face any singularity if one extrapolates the α' metric from infinity toward the origin.

2. One can choose the boundary conditions requiring a smooth geometry . A boundary condition which is different from (2.22) results in

$$ds^2 = dt^2 + \tanh(t)^2 \left(1 + \alpha' C_1 + \alpha'^2 C_2 - \frac{2\alpha'}{\cosh(t)^2} - \frac{\alpha'^2(5-\cosh(t))}{\cosh(t)^4}\right) dr^2\,,$$

   where C_1 and C_2 are arbitrary real numbers. These numbers can be chosen in such a way that the metric remains nonzero. This choice alters the metric at infinity.

3. One can employ the field redefinition ambiguity to obtain a regular metric. For example redefining the metric by

$$g_{\mu\nu} \to g_{\mu\nu} + \alpha'\, R_{\mu\nu} + \alpha'^2 \left(\frac{1}{2} R_{\mu\eta} R^{\eta}{}_{\nu} + \frac{3}{8} R\, R_{\mu\nu} - \frac{1}{4}\Box R_{\mu\nu} + \frac{3}{8}\nabla_\mu\nabla_\nu R\right),$$

   gives

$$ds^2 = \left(1 + \frac{2\alpha'}{\cosh(t)^2} + \frac{4\alpha'^2}{\cosh(t)^2}\right) dt^2 + \tanh(t)^2\, dr^2\,, \tag{3.3}$$

   where it is clear that the geometry of the redefined metric is smooth. One may ask if the β functions of such a redefined metric have better properties. For example, do all the exact modes of the linearized action have real frequency [15]?, or do the beta functions admit covariant divergence free currents [17]? These questions need further investigation.

4. We can rewrite the α' expansion series in such a way that the components of the metric remain nonzero. For example [29] uses a different series which gives a regular metric. Note that as shown in [29] the linear α' corrections are the same as perturbative expansion of [22] in which a modification of the two-dimensional black hole is conjectured to be α'-exact while the agreement at the quadratic order requires a field redefinition.

Also a combination of the above methods can be chosen to obtain a regular α' corrected background.

4. Conclusions and discussions

In the second section we computed the quadratic α' corrections to the two-dimensional black hole and to its T-dual. These α'^2 corrected backgrounds and the α'^2 corrected Kasner metric [12] were used to write the covariant form of the α'^2 corrections to T-duality for general time-dependent background of diagonal metric and dilaton in bosonic string theory. It was shown that no appropriate perturbative field redefinition compensates the corrections to T-duality. Thus tree level rules of T-duality must be modified. This argument was supported by the fact that T-duality rules are modified for the conjectured α' exact backgrounds [22].

In the third section we showed that the α' correction to the string coupling constant is finite for the two-dimensional black hole. Therefore the string loop effects can be ignored consistently not only at the tree level [23] but also to, at least, the quadratic order in α'. It was observed that the α' corrections may change the horizon to an intrinsic singularity. However no concrete conclusion was achieved due to the fact that the curvature at the horizon is comparable to the string length and one must take into account all the α' corrections. Note that the perfect reflection of the tachyon wave [23] points to the possibility of the absence of a horizon, while the conjectured α' exact background [22] supports the existence of the horizon.

We presented four different methods to preserve the horizon in the perturbation theory. In addition to these possibilities other fields may be required to be turned on besides the field content of the tree level background (2.18). To clarify this statement recall that the subleading corrections to the β function of the dilaton can induce a non-trivial dilaton at the subleading level when the tree level dilaton vanishes. A priori it is not clear if all other modes, in particular the massless tachyon, can be consistently set to zero to all orders in α'. In the context of WZW coset models, the conjectured α'-exact two-dimensional black hole [22] has only non trivial dilaton and metric. However in the context of MQM ref. [30] argues that certain winding modes of Tachyon should be turned on for the two-dimensional black hole, this possibility is also emphasized in [23, 31].

It is a hopeless task to calculate the α' perturbative expansion of the beta functions of a general background, a background which includes all the excitation of string theory. It is easier to guess the field contents of the underlying conformal theory than to ask if the corresponding beta functions have a perturbative solution. When the answer is affirmative, like the case of the two-dimensional black hole, the guess can be correct.

Acknowledgments

I thank Loriano Bonora and Martin O'Loughlin for both the suggestions and the discussions throughout the work, and Carlo Maccaferri for discussions. I thank Ricardo Schiappa and Daniel Grumiller for the correspondences.

A. The α'^2 corrections to the Schwarzschild metric in $D = 5$

The five dimensional Schwarzschild black hole reads

$$ds^2 = -\left(1 - \frac{1}{r^2}\right) dt^2 + \frac{dr^2}{1 - \frac{1}{r^2}} + r^2\, d\Omega_3\,, \tag{A.1}$$

where the mass of the black hole is appropriately chosen to give a factor of one in $\frac{1}{r^2}$. For very large r this metric can be generalized to a perturbative background of free critical bosonic string theory,[8]

$$\begin{aligned}
ds^2 &= -\, f(r)\, dt^2 + g(r)\, dr^2 + r^2\, d\Omega_3 + \overrightarrow{dx}_{21}{}^2\,, && \text{(A.2)}\\
f(r) &= \left(1 - \frac{1}{r^2}\right)(1 + \alpha'\, f^{(1)}(r) + \alpha'^2\, f^{(2)}(r) + \cdots)\,, && \text{(A.3)}\\
g(r) &= \frac{1}{1 - 1/r^2}(1 + \alpha'\, g^{(1)}(r) + \alpha'^2\, g^{(2)}(r) + \cdots)\,, && \text{(A.4)}\\
\phi(r) &= 0 + \alpha'\, \phi^{(1)}(r) + \alpha'^2\, \phi^{(2)}(r) + \cdots\,, && \text{(A.5)}
\end{aligned}$$

where $\overrightarrow{dx}_{21}$ represents the 21 flat directions compactified to a torus and $\phi(r)$ is the dilaton. Using the beta functions and assuming that there is no correction to the mass of the black hole or to the fall off of the dilaton at infinity, one finds

$$\begin{aligned}
\phi^{(1)}(r) &= \frac{25}{8} \ln\left(\frac{r^2+1}{r^2-1}\right) - \frac{25}{12\, r^6} - \frac{25}{4\, r^2}\,, && \text{(A.6)}\\
f^{(1)}(r) &= \frac{1}{6\, r^6\, (r^4-1)} \left(48 + 80r^4 - 30r^8 + 15r^6(r^4-3) \ln\left(\frac{r^2+1}{r^2-1}\right)\right)\,, && \text{(A.7)}\\
g^{(1)}(r) &= \frac{1}{3r^6(r^4-1)} \left(11 - 30r^4 + 15 \ln\left(\frac{r^2+1}{r^2-1}\right)\right)\,, && \text{(A.8)}
\end{aligned}$$

[8]The linear α' corrections to the four dimensional black hole were computed in [32].

and

$$\phi^{(2)}(r) = \frac{2364r^8 - 2478r^6 - 394r^4 - 953r^2 - 915}{768\,r^8\,(r^2-1)} + \tag{A.9}$$

$$+\frac{\ln(1-\frac{1}{r^2})}{64r^4(r^2-1)}(197r^6 - 305r^4 + 54r^2 - 54)\,,$$

$$f^{(2)}(r) = \frac{1068\,r^{10} - 3966r^8 + 1214\,r^6 - 435\,r^4 - 1553r^2 + 6048}{576\,(r^2-1)\,r^{10}} + \tag{A.10}$$

$$+\frac{\ln(1-\frac{1}{r^2})}{48\,r^{12}\,(r^2-1)^2}\,(89\,r^8 - 321\,r^6 + 16\,r^4 - 108\,r^2 + 432) +$$

$$+\frac{9\,(r^2-4)}{8}\left(\ln\left(1-\frac{1}{r^2}\right)\right)^2,$$

$$g^{(2)}(r) = \frac{876\,r^8 + 210\,r^6 + 574\,r^4 + 6449\,r^2 - 4473}{576\,r^6\,(r^2-1)^3} + \tag{A.11}$$

$$-\frac{(73\,r^4 - 73\,r^2 + 144)}{48\,(r^2-1)^3}\ln\left(1-\frac{1}{r^2}\right) + \frac{9\,r^2\,(r^2+1)}{8\,(r^2-1)^3}\ln\left(1-\frac{1}{r^2}\right)^2.$$

Note that the α' corrected metric has a singularity outside the horizon. This is reminiscent of what happens in the case of two-dimensional black hole. In addition the α' corrected string coupling constant diverges at $r = 1$. We come back to these issues at the end of this appendix. Now let us apply T-duality in the direction of the time-like Killing vector outside the horizon

$$d\tilde{s}^2 = -\frac{dt^2}{1-1/r^2} + \frac{dr^2}{1-1/r^2} + r^2\,d\Omega_3\,, \tag{A.12}$$

$$\tilde{\phi}(r) = 0\,. \tag{A.13}$$

The T-dual of the Schwarzschild metric is singular at "$r = 1$" which means that the coordinate singularity of the Schwarzschild metric has changed to an intrinsic singularity. Since T-duality relates the singular time-dual geometry to the Schwarzschild metric then one expects that the time-dual metric to be as stable as the Schwarzschild metric [33]. This argument should not sound strange because the classical stability of a naked singularity recently has been explored in [34]. At large r T-dual background can be generalized to a perturbative background of string theory whose quadratic α' corrections follow [9]

$$d\tilde{s}^2 = -\,\tilde{f}(r)\,dt^2 + g(r)\,dr^2 + r^2\left(1+\frac{\alpha'^2}{r^8\,(r^2-1)}\right)d\Omega_3 + \overrightarrow{dx}_{21}{}^2\,, \tag{A.14}$$

$$\tilde{f}(r) = \frac{1}{1-1/r^2}\,(1+\alpha'\,f^{(1)}(r) + \alpha'^2\,f^{(2)}(r) + \cdots)\,, \tag{A.15}$$

$$\tilde{\phi}(r) = 0 + \alpha'\,\tilde{\phi}^{(1)}(r) + \alpha'^2\,\tilde{\phi}^{(2)}(r) + \cdots\,, \tag{A.16}$$

where we are going to assume that there is no α'-correction to the fall off of $f(r)$ at infinity. Note that $g(r)$ is provided by (A.4). The linear and the quadratic terms in α' in $\tilde{f}(r)$ follow

$$\tilde{f}^{(1)}(r) = \frac{6\,r^2 - 9}{4\,(r^2-1)\,r^2} + \frac{3\,(r^2-2)}{2\,(r^2-1)}\ln\left(1-\frac{1}{r^2}\right), \tag{A.17}$$

[9] We have chosen this specific coordinate since it is easier to apply T-duality in this coordinate.

$$\tilde{f}^{(2)}(r) = -\frac{1068\,r^{10} - 5262\,r^8 + 5102\,r^6 - 1623\,r^4 + 1039\,r^2 + 576}{576\,r^8\,(r^2-1)^2} + \\ -\frac{89\,r^4 - 448\,r^2 + 324}{48r^2(r^2-1)}\ln\left(1-\frac{1}{r^2}\right) + \frac{9(r^4-3r^2+4)}{8(r^2-1)^2}\left(\ln\left(1-\frac{1}{r^2}\right)\right)^2. \quad (A.18)$$

And the coefficients of α' in $\tilde{\phi}(r)$ read

$$\tilde{\phi}^{(1)}(r) = -\frac{6\,r^4 + 9\,r^2 - 1}{16\,r^4\,(r^2-1)} - \frac{3\,(r^2+1)}{8\,(r^2-1)}\ln\left(1-\frac{1}{r^2}\right), \quad (A.19)$$

$$\tilde{\phi}^{(2)}(r) = \frac{4956r^{10} - 5298r^8 - 64r^6 - 1347r^4 + 1492r^2 - 2151}{2304(r^2-1)^2r^8} + \\ +\frac{413r^8 - 648r^6 + 289r^4 - 36r^2 + 18}{194\,r^4\,(r^2-1)}\ln\left(1-\frac{1}{r^2}\right) + \\ +\frac{9\,r^2}{16\,(r^2-1)^2}\left(\ln\left(1-\frac{1}{r^2}\right)\right)^2. \quad (A.20)$$

Instead of writing the α' corrected Schwarzschild black hole and its T-dual metric in the "radial" co-moving frame, it is easier to rewrite the rules of T-duality for metrics which can be transform to the "radial" co-moving frame by the same coordinate transformation. Here the α' corrected Schwarzschild black hole and its T-dual can be transformed to the "radial" co-moving frame by the same coordinate transformation. Luckily this transformation does not alter the rules of T-duality. It is a straightforward computation to check that the α' corrected T-duality rules (2.3), (2.4), (2.32) relate the α' corrected black-hole to its T-dual background.

The α' corrected metric is singular outside the horizon. If one insists on having a smooth geometry then one can choose the boundary conditions by requiring finite corrections at $r = 1$,

$$ds^2 = -f^\star(r)\left(1-\frac{1}{r^2}\right)dt^2 + \frac{g^\star(r)}{1-1/r^2}\,dr^2 + r^2\,d\Omega_3^2\,,$$
$$f^\star(r) = 1 + c_1\alpha' + c_2\alpha'^2 - \alpha'\frac{17r^2+8}{4r^4} - \alpha'^2\frac{1039\,r^6 - 4811\,r^4 - 10543\,r^2 - 6048}{576\,r^8}\,,$$
$$g^\star(r) = 1 - \alpha'\,\frac{r^2+7}{4\,r^4} + \alpha'^2\,\frac{355\,r^6 - 1565\,r^4 + 2497\,r^2 + 4473}{576\,r^8}\,,$$
$$\phi^\star(r) = 0 - \alpha'\frac{9\,(1+2\,r^2)}{16\,r^4} - \alpha'^2\frac{228\,r^6 + 6\,r^4 - 1868\,r^2 - 915}{768\,r^8}\,, \quad (A.21)$$

where c_1 and c_2 are numerical constants. These boundary conditions also gives finite corrections to the string coupling constant. One may fix "$c_1 = c_2 = 0$" assuming no correction exist at infinity. The price of having a smooth geometry is to change the fall off of the metric at infinity. The asymptotic behaviour of the linear α' corrected metric at large r in the Einstein frame is

$$ds_E^{\star 2} = \left(1 + 2c_1\alpha' - \frac{1+\frac{11}{4}\alpha'}{r^2}\right)dt^2 + \left(1 + \frac{1+\frac{5}{4}\alpha'}{r^2}\right)dr^2 + r^2\,d\Omega_3^2 + O(\alpha'^2) + O\left(\frac{1}{r^4}\right).$$

The corrections to the T-dual background are

$$
\begin{aligned}
d\tilde{s}^{\star 2} &= -\frac{\tilde{f}^{\star}(r)}{1-1/r^2}\,dt^2 + \frac{g^{\star}(r)}{1-1/r^2}\,dr^2 + r^2\left(1+\frac{\alpha'^2}{r^8\,(r^2-1)}\right)d\Omega_3^2\,,\\
\tilde{f}^{\star}(r) &= 1-\alpha'\frac{17\,r^2-9}{4\,r^2\,(r^2-1)} + \\
&\quad -\alpha'^2\,\frac{-576+1623\,r^4-1039\,r^2+3515\,r^8-1106\,r^6+1039\,r^{10}}{576\,r^8\,(r^2-1)^2}\,,\\
\tilde{\phi}^{\star}(r) &= -\frac{1}{2}\ln\left(1-\frac{1}{r^2}\right)+\alpha'\frac{16\,r^4-9\,r^2+1}{16\,r^4\,(r^2-1)} + \\
&\quad +\alpha'^2\,\frac{-1527\,r^4+1492\,r^2-2024\,r^8+3968\,r^6+1394\,r^{10}-2151}{2304\,r^8\,(r^2-1)^2}\,.
\end{aligned}
\tag{A.22}
$$

These expressions are consistent with α' corrected T-duality. We could have used T-duality to find the corrections, by the same method that [35] finds the linear α' corrections to the T-dual of the four dimensional black hole.

References

[1] K. Kikkawa and M. Yamasaki, *Casimir effects in superstring theories*, *Phys. Lett.* **B 149** (1984) 357;
N. Sakai and I. Senda, *Vacuum energies of string compactified on torus*, *Prog. Theor. Phys.* **75** (1986) 692.

[2] T.H. Buscher, *Path integral derivation of quantum duality in nonlinear sigma models*, *Phys. Lett.* **B 201** (1988) 466.

[3] A.A. Tseytlin, *Duality and dilaton*, *Mod. Phys. Lett.* **A 6** (1991) 1721.

[4] P.E. Haagensen and K. Olsen, *T-duality and two-loop renormalization flows*, *Nucl. Phys.* **B 504** (1997) 326 [hep-th/9704157];
P.E. Haagensen, K. Olsen and R. Schiappa, *Two-loop beta functions without feynman diagrams*, *Phys. Rev. Lett.* **79** (1997) 3573 [hep-th/9705105].

[5] N. Kaloper and K.A. Meissner, *Duality beyond the first loop*, *Phys. Rev.* **D 56** (1997) 7940 [hep-th/9705193].

[6] K. Olsen and R. Schiappa, *Heterotic T-duality and the renormalization group*, *Int. J. Mod. Phys.* **A 14** (1999) 2257 [hep-th/9805074].

[7] K.A. Meissner and G. Veneziano, *Symmetries of cosmological superstring vacua*, *Phys. Lett.* **B 267** (1991) 33;
Manifestly $O(d,d)$ invariant approach to space-time dependent string vacua, *Mod. Phys. Lett.* **A 6** (1991) 3397 [hep-th/9110004].

[8] A. Sen, *$O(d)\otimes O(d)$ symmetry of the space of cosmological solutions in string theory, scale factor duality, and two-dimensional black holes*, *Phys. Lett.* **B 271** (1991) 295.

[9] S.F. Hassan and A. Sen, *Twisting classical solutions in heterotic string theory*, *Nucl. Phys.* **B 375** (1992) 103 [hep-th/9109038].

[10] K.A. Meissner, *Symmetries of higher-order string gravity actions*, *Phys. Lett.* **B 392** (1997) 298 [hep-th/9610131].

[11] I. Jack and S. Parsons, *$O(d,d)$ invariance at two and three loops*, *Phys. Rev.* **D 62** (2000) 026003 [hep-th/9911064].

[12] G. Exirifard and M. O'Loughlin, *Two and three loop α' corrections to T-duality: Kasner and Schwarzschild*, *JHEP* **12** (2004) 023 [hep-th/0408200].

[13] I. Jack, D.R.T. Jones and D.A. Ross, *The four-loop dilaton beta-function*, *Nucl. Phys.* **B 307** (1988) 531.

[14] A.A. Tseytlin, *Conformal anomaly in two-dimensional sigma model on curved background and strings*, *Phys. Lett.* **B 178** (1986) 34; *Sigma model Weyl invariance conditions and string equations of motion*, *Nucl. Phys.* **B 294** (1987) 383.

[15] A. Sen, *How does a fundamental string stretch its horizon?*, *JHEP* **05** (2005) 059 [hep-th/0411255].

[16] V. Hubeny, A. Maloney and M. Rangamani, *String-corrected black holes*, *JHEP* **05** (2005) 035 [hep-th/0411272].

[17] D. Lovelock, *The Einstein tensor and its generalizations*, *J. Math. Phys.* **12** (1971) 498.

[18] P.E. Haagensen, *Duality and the renormalization group*, hep-th/9708110.

[19] M. Mueller, *Rolling radii and a time dependent dilaton*, *Nucl. Phys.* **B 337** (1990) 37.

[20] G. Mandal, A.M. Sengupta and S.R. Wadia, *Classical solutions of two-dimensional string theory*, *Mod. Phys. Lett.* **A 6** (1991) 1685.

[21] E. Witten, *On string theory and black holes*, *Phys. Rev.* **D 44** (1991) 314.

[22] R. Dijkgraaf, H. Verlinde and E. Verlinde, *String propagation in black hole geometry*, *Nucl. Phys.* **B 371** (1992) 269.

[23] J.L. Karczmarek, J. Maldacena and A. Strominger, *Black hole non-formation in the matrix model*, hep-th/0411174.

[24] A. Buchel, *Coarse-graining 1/2 BPS geometries of type-IIB supergravity*, hep-th/0409271.

[25] H. Lin, O. Lunin and J. Maldacena, *Bubbling AdS space and 1/2 BPS geometries*, *JHEP* **10** (2004) 025 [hep-th/0409174].

[26] R.G. Leigh, *Dirac-Born-Infeld action from dirichlet sigma model*, *Mod. Phys. Lett.* **A 4** (1989) 2767.

[27] M.N.R. Wohlfarth, *Gravity à la Born-Infeld*, *Class. and Quant. Grav.* **21** (2004) 1927 [hep-th/0310067].

[28] D. Grumiller, *An action for the exact string black hole*, *JHEP* **05** (2005) 028 [hep-th/0501208].

[29] A.A. Tseytlin, *On the form of the black hole solution in $D = 2$ theory*, *Phys. Lett.* **B 268** (1991) 175.

[30] V. Kazakov, I.K. Kostov and D. Kutasov, *A matrix model for the two-dimensional black hole*, *Nucl. Phys.* **B 622** (2002) 141 [hep-th/0101011].

[31] A. Giveon, D. Kutasov, E. Rabinovici and A. Sever, *Phases of quantum gravity in AdS_3 and linear dilaton backgrounds*, *Nucl. Phys.* **B 719** (2005) 3 [hep-th/0503121].

[32] C.G. Callan Jr., R.C. Myers and M.J. Perry, *Black holes in string theory*, *Nucl. Phys.* **B 311** (1989) 673.

[33] A. Ishibashi and H. Kodama, *Stability of higher-dimensional Schwarzschild black holes*, *Prog. Theor. Phys.* **110** (2003) 901 [hep-th/0305185].

[34] G.W. Gibbons, S.A. Hartnoll and A. Ishibashi, *On the stability of naked singularities*, *Prog. Theor. Phys.* **113** (2005) 963 [hep-th/0409307].

[35] C.P. Burgess, R.C. Myers and F. Quevedo, *Duality and four-dimensional black holes*, *Nucl. Phys.* **B 442** (1995) 97 [hep-th/9411195].